

Nonconvex Adjustment Costs in Lumpy Investment Models: Mean versus Variance

Min Fang

University of Rochester

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- ▶ Unknown: Expected Size (Mean) vs Uncertainty (Variance),

which one determines the interest elasticity of aggregate investment?

This paper

- ▶ **Theory:** Separate the roles of mean and variance in lumpy investment models:

$$\xi_{jt} \sim U[\mu_\xi - \sqrt{3}\sigma_\xi, \mu_\xi + \sqrt{3}\sigma_\xi] \quad (1)$$

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$$\xi_{jt} \sim U[\mu_{\xi} - \sqrt{3}\sigma_{\xi}, \mu_{\xi} + \sqrt{3}\sigma_{\xi}] \quad (1)$$

- ▶ **Results:** Both mean and variance matter for the interest elasticity of agg. inv.

$$\frac{d \sum I_j}{dr} = \frac{d \sum_{EM} I_j}{dr} + \frac{d \sum_{IM} I_j}{dr}$$

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$$\frac{d \sum I_j}{dr} = \frac{d \sum_{EM} I_j}{dr} + \frac{d \sum_{IM} I_j}{dr}$$

1. Expected Size (*Mean*) governs the importance of the extensive margin:
2. Uncertainty (*Variance*) governs the sensitivity of the extensive margin:

$$\text{larger } \mu_{\xi} \Rightarrow \text{larger } \frac{d \sum_{EM} I_j}{dr} / \frac{d \sum I_j}{dr} \quad \text{larger } \sigma_{\xi} \Rightarrow \text{smaller } \frac{d \sum_{EM} I_j}{dr}$$

Outline of today's talk

- ▶ A quick revisit of the lumpy investment models
- ▶ Show interest elasticity of agg. inv. along calibrations of the bundled model
- ▶ Show interest elasticity of agg. inv. along calibrations of the separate models
- ▶ Show mechanism of mean and variance in decomposition of EM and IM
- ▶ Show mechanism of mean and variance in distributions of EM and IM
- ▶ Conclusion and future research

The model: setup

Technology:

$$y_{jt} = z_{jt} k_{jt}^{\alpha} n_{jt}^{\nu}, \quad \alpha + \nu < 1 \quad (2)$$

$$\log(z_{jt}) = -(1 - \rho^z) \frac{\sigma^z}{2(1 - \rho^{z2})} + \rho^z \log(z_{jt-1}) + \epsilon_{jt}, \quad \epsilon_{jt} \sim N(0, \sigma^z) \quad (3)$$

Adjustment Costs:

$$c(i_{jt}) = i_{jt} + \mathbf{1}_{(|i_{jt}| > ak_{jt})} \cdot w_t \cdot \xi_{jt}, \quad \xi_{jt} \sim \text{eq.}(1) \quad (4)$$

Value Functions:

$$V^A(k_{jt}, z_{jt}; \Omega_t) = \max_{i,n} \left\{ y_{jt} - w_t n_{jt} - c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k_{jt+1}^*, z_{jt+1}; \Omega_{t+1})] \right\} \quad (5)$$

$$V^{NA}(k_{jt}, z_{jt}; \Omega_t) = \max_{i \in [-ak, ak], n} \left\{ y_{jt} - w_t n_{jt} - c(i_{jt}) + \mathbb{E}[\Lambda_{t,t+1} V(k_{jt+1}^C, z_{jt+1}; \Omega_{t+1})] \right\} \quad (6)$$

The model: optimization

Value Function:

$$V(k_{jt}, z_{jt}; \Omega_t) = -\frac{w_t(\xi^* + \xi)}{2} + \frac{\xi^* - \xi}{2\sqrt{3}\sigma_\xi} V^A(k_{jt}, z_{jt}; \Omega_t) + \left(1 - \frac{\xi^* - \xi}{2\sqrt{3}\sigma_\xi}\right) V^{NA}(k_{jt}, z_{jt}; \Omega_t) \quad (7)$$

Extensive Margin:

$$\xi_t^*(k_{jt}, z_{jt}; \Omega_t) = \frac{V^A(k_{jt}, z_{jt}; \Omega_t) - V^{NA}(k_{jt}, z_{jt}; \Omega_t)}{w_t} \quad (8)$$

Intensive Margin:

$$k_{jt+1} = \begin{cases} (1 - \delta)k_{jt} + i_{jt}^* & \xi_{jt} < \xi^*(k_{jt}, z_{jt}; \Omega_t) \\ (1 - \delta)k_{jt} + i_{jt}^C & \text{otherwise} \end{cases} \quad (9)$$

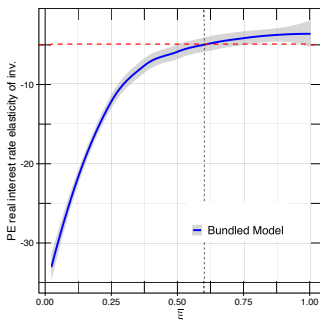
There is also a representative household to complete the GE model.

The calibration of parameters is very standard as in the literature.

Interest elasticity of the bundled model

- The benchmark is calibrated $\mu_\xi = \sqrt{3}\sigma_\xi = \bar{\xi}/2$, a $\bar{\xi} = 0.6 \Rightarrow \frac{d \sum I_j}{dr} = -5$

Figure: Interest Elasticity over $\bar{\xi}$

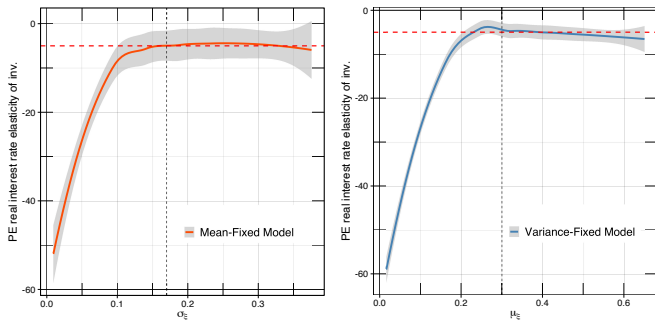


Note: In the benchmark model, the uniform distribution starts from 0 to an upper bound: $\xi_{jt} \sim U[0, \bar{\xi}]$. It bundles the mean and the variance by $\bar{\xi}$: $\mu_\xi = \sqrt{3}\sigma_\xi = \bar{\xi}/2$. Therefore, increasing $\bar{\xi}$ increases both μ_ξ and σ_ξ simultaneously.

Interest elasticity of the separate models

- ▶ The Mean-Fixed Model fixed $\mu_\xi = \bar{\xi}/2$, varying σ_ξ
- ▶ The Variance-Fixed Model fixed $\sigma_\xi = \bar{\xi}/\sqrt{12}$, varying μ_ξ

Figure: Interest-Elasticity over μ_ξ and σ_ξ



(a) $\mu_\xi^* = \bar{\xi}/2$, change σ_ξ

(b) $\sigma_\xi^* = \bar{\xi}/\sqrt{12}$, change μ_ξ

Note: The variance-fixed model fixes the variance by choosing $\sigma_\xi^* = \bar{\xi}/\sqrt{12}$ and the mean-fixed model fixes the mean by choosing $\mu_\xi^* = \bar{\xi}/2$. The two models are identical along both vertical dotted lines when $\mu_\xi = 0.3$ and $\sigma_\xi \simeq 0.17$.

A decomposition of the extreme calibrations

- ▶ 1).the carefully calibrated bundled model (*Bundled*);
- ▶ 2).a mean-fixed model ($\mu_{\xi}^* = \bar{\xi}/\sqrt{12}$) with zero variance (*Zero- σ_{ξ}*);
- ▶ 3).a variance-fixed model ($\sigma_{\xi}^* = \bar{\xi}/2$) with zero mean (*Zero- μ_{ξ}*).

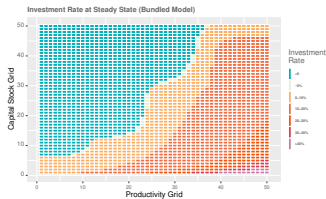
Table: A Decomposition of the Interest Elasticity

Decomp.	Bundled			Zero- σ_{ξ}			Zero- μ_{ξ}		
	Total	EM	IM	Total	EM	IM	Total	EM	IM
Elasticity	-5.1	-4.9	-0.2	-601.8	-601.7	-0.1	-80.0	-4.4	-75.6
Percentage	—	96%	4%	—	100%	0%	—	5.5%	94.5%

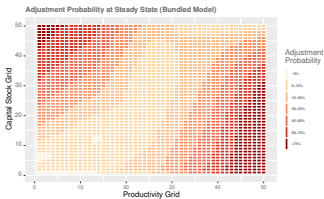
Note: The *Bundled* model has the calibration that matches the micro investment moments. The Zero- σ_{ξ} deviates by setting σ_{ξ} to zero while all other parameters are unchanged. The Zero- μ_{ξ} deviates by setting μ_{ξ} to zero while all other parameters are unchanged. EM stands for the extensive margin and IM stands for the intensive margin.

Distributions of EM & IM in the extreme calibrations

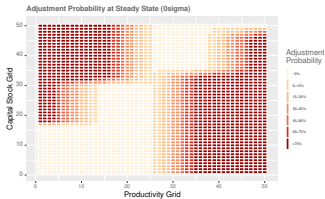
Figure: Distributions of the Extensive Margin and the Intensive Margin



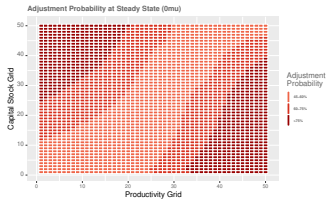
(a) Intensive Margin (Bundled)



(b) Extensive Margin (Bundled)



(c) Extensive Margin (Zero- σ_{ξ})



(d) Extensive Margin (Zero- μ_{ξ})

Conclusion

Both mean and variance matter for the interest elasticity of agg. inv.:

- ▶ The mean governs the importance of the extensive margin.
- ▶ The variance governs the sensitivity of the extensive margin.

Future research:

- ▶ more realistic estimations of the mean and the variance using microdata on firm-level investment.
- ▶ the separate roles of the mean and the variance potentially matter for other dynamic models with such as firm entry and exit, worker hire and fire, trade entry and exit, inventory dynamics, and many others.